A Synopsis On

A STUDY ON PREFUNCTIONS AND INTEGRAL EQUATIONS

Submitted to

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For the Degree of

Doctor of Philosophy (Ph.D.)

In Mathematics (Faculty of Science and Technology)

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September - 2022

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1. Introduction:

J. Fourier (1768-1830) is the initiator of the theory of integral equations. A term integral equation first suggested by Du Bois-Reymond in 1888. Du Bois-Reymond define an integral equation is understood an equation in which the unknown function occurs under one or more signs of definite integration. Late eighteenth and early ninetieth century Laplace, Fourier, Poission, Liouville and Abel [1-3] studies some special type of integral equation. The pioneering systematic investigations goes back to late nineteenth and early twentieth century work of volterra, Fredholm and Hilbert. In 1887, Volterra published a series of famous papers in which he singled out the notion of a functional and pioneered in the development of a theory of functional s in theory of linear integral equation of special type. Fredholm presented the fundamentals of the Fredholm integral equation theory in a paper published in 1903 in the Acta Mathematica. This paper became famous almost overnight and soon took its rightful place among the gems of modern mathematics. Hilbert followed Fredholm's famous paper with a series of papers in the Nachrichten of the GUttingen Academy. Danish-Ul-Islam & Irfan Ul Haq [4] was studied the solution of the Fredholm Integral Equations by using Analytic Method. Many Researchers [5] studied many integral equations with solutions as well as analytical and numerical methods for solving linear and nonlinear equations. It explores Volterra, Fredholm, Weiner-Hof, Hammerstein, Uryson and other equations that arise in Mthemtics, Physics, Engineering, The sciences and Economics. Kendall E. Atkinson [6] have studied numerical methods for calculating fixed points of nonlinear integral operators that are applicable in variety of nonlinear integral equations. Saha and Sahu [7] have review various numerical techniques for solving linear and nonlinear Fredholm integral equations of second kind.

(a) Integral equation

Integral equation is an equation in which the unknown, say a function of a numerical variable, occurs under an integral. That means a functional equation involving the unknown function under one or more integrals. For example,

$y(x) = f(x) + \int_0^1 K(x,t)\, y(t)dt$, where k(x,t) is kernel of integral equations. Further it is classified into Separated Kernel, Symmetric Kernel, Resolvent or Reciprocal kernel, Difference Kernel etc.

(b) Prefunctions and Extended Prefunctions:

The pre-exponential functions is denoted by $pexp(t,\alpha)$ and is defined as,

$$pexp(t,\alpha) = 1 + \sum_{n=1}^{\infty} \frac{t^{n+\alpha}}{(n+1+\alpha)}$$

For any real number t and α being a parameter. Here $pexp(t,\alpha)$ for each $\alpha \geq 0$ stands for prefunction of $exp(t)$.

The Pretrigonometric Functions (Pre-cosine function and Pre-Sine) is denoted by $psin(t,\alpha)$, $pcos(t,\alpha)$ respectively and is defined as

$$psin(t,\alpha) = \sum_{n=0}^{\infty}(-1)^n \frac{t^{2n+1+\alpha}}{(n+\ +\alpha)}, \qquad \alpha \geq 0, tcR$$

$$pcos(t,\alpha) = 1 + \sum_{n=1}^{\infty}(-1)^n \frac{t^{2n+\alpha}}{(n+1+\alpha)}, \qquad \alpha \geq 0, tcR$$

The Extended Prefunction is defined as

$$pM_{3,2}(t,\alpha) = \sum_{n=0}^{\infty}(-1)^n \frac{t^{3n+2+\alpha}}{(n+\ +\alpha)}, \qquad \alpha \geq 0, tcR$$

2. Review of Literature and development in the subject:

In 2010 Khandeparkar, Deo and Dhaigude [8-9], have defined new kind of Exponential, Hyperbolic and Trigonometric functions, they are termed as Prefunctions and extended Prefunctions. It has been shown that, these pre-functions and extended Prefunctions satisfy many interesting properties and relations. Some properties and Laplace Transforms of these functions are studied and solutions of initial value problems for non- homogeneous linear ordinary differential equations, system of Differential Equations via Laplace transform, Voltera integral equations, Voltera integro- differential equations obtained [10-12]. A.P. Hiwarekar [13] have studied pre-trigonometric and pre-hyperbolic functions and their relation with Mittag-Leffler function which plays very important role in fractional calculus.

3. Aim and Objectives of Research:

We will work for a unified understanding of basic concept of Prefunctions and extended Prefunctions. Also we will be study some integral equations of prefunctions and their applications.

4. Methodology:

Relevant knowledge of the concern topic will be obtained by the survey of research papers, review articles, various books and journals. Also we would like to visit different research institutes like BAMU, Aurangabad, Goa University, Goa etc. to interact with eminent Mathematicians also would like to attend national, international conference and Workshop related to our research work.

5. Importance of study:

The study of integral equations plays an important role because the study finds applications in Science, Engineering and Mathematical Economics. In the present work we will study some integral equations and prefunctions with modern Mathematical software's like MATLAB, SCILAB and Python etc.

6. Plan of Research Work:

Sr.No	Period	Plan of the work	Targets to be achieved
1	First year	Study of the background required for the research topic.	Good command on the subject
2	Second year	Collecting the papers and analyse them.	Try to get some Construction techniques.
3	Third Year	Work on some problems to be included in the thesis.	Submit at least two papers.
4	Fourth Year	Publishing some papers and submitting the Thesis.	Some more papers and Thesis submission.

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